

## **ADDENDUM TO THE RICHARDSON MATHFORWARD PROGRAM ANALYSIS REPORT**

### **INTRODUCTION**

The original Richardson MathForward Program analysis for 2005 – 2007 is reviewed here and supplemented with a revised analysis to account for certain assumptions not addressed previously. While the general conclusions have not changed from the original report, they are expanded upon here with revised values that are believed to be a more accurate representation of what was happening to students in the study between 2005 and 2007.

This addendum report will use the same taxonomy of the original report:

- Study – students who were in the MathForward program
- Control 1 – students who were taught by MathForward teachers but were not in the program
- Control 2 – students who were taught by teachers not in the MathForward program but were at the same schools that the program was implemented
- Control 3 – students who attended schools where the program was not implemented

However, instead of using the transformed normal curve equivalent (TNCE) values from the original report, all TAKS Math scale scores will be reported as a standardized z-score that were determined using values reported by TEA. The rationale for this is that the TNCE were all relative to the “Met Standards” performance rating of the TAKS exam. The rating itself is variable from grade to grade, making cross grade comparisons unsound and yields no information on how students performed relative to each other. The standard score, on the other hand, would yield information on students’ relative performance to the testing population.

In the original analysis, Ordinary Least Squares (OLS) regression was done as is common practice in the educational research literature. However, issues concerning violations of the basic assumptions of OLS were raised; namely, that the independent variable should be error free. The case under examination was a repeated measures study that would have

been more suited to an error-in-variables model such as Reduced Major Axis (RMA) regression.

The use of RMA regression in this addendum report means that Regression Discontinuity could not be performed due to the nature of how RMA regression determines the best fit line. OLS regression minimizes the squares perpendicular to the x-axis. RMA regression minimizes the squares perpendicular to the best fit line. Regression Discontinuity depends on the usage of an x-axis cutoff value and would yield biased values when using RMA regression due to uneven distribution perpendicular to the best fit line at the cutoff value.

## RESULTS

### *Descriptive Statistics and ANOVA*

Since TNCE were not used in this addendum report, descriptive statistics were determined again using the standard scores as shown in Table 1. The difference between the 2007 and 2006 TAKS Math standard score was represented by the symbol  $\Delta$ .

	2006	2007	$\Delta$
<b>Study</b>	-0.7060	-0.3693	0.3368
<b>Control 1</b>	-0.4285	-0.4372	-0.0087
<b>Control 2</b>	-0.0027	0.0562	0.0590
<b>Control 3</b>	-0.0097	-0.0130	-0.0033

**Table 1. Mean standard TAKS Math scores of the different groups**

An ANOVA test was done to determine if there were significant differences between the  $\Delta$  of the different groups as shown in Table 2. The dependent variable was  $\Delta$  and the independent variable was Group membership.

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	32.469(a)	3	10.823	36.059	.000
Intercept	7.585	1	7.585	25.270	.000
Group	32.469	3	10.823	36.059	.000
Error	678.040	2259	.300		
Total	722.521	2263			
Corrected Total	710.510	2262			

a R Squared = .046 (Adjusted R Squared = .044)

**Table 2. ANOVA of the different groups to determine significant differences**

Since significant differences were found, a Tukey HSD post hoc analysis was done to determine which of the groups were different as shown in Table 3. The coding system was 0, 1, 2, 3 for the Study, Control 1, Control 2, and Control 3 groups respectively.

(I) group	(J) group	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
0	1	<b>0.3451*</b>	0.07141	0.000	0.1615	0.5287
	2	<b>0.2778*</b>	0.03464	0.000	0.1887	0.3669
	3	<b>0.3400*</b>	0.03312	0.000	0.2549	0.4252
1	0	<b>-0.3451*</b>	0.07141	0.000	-0.5287	-0.1615
	2	-0.0673	0.06839	0.759	-0.2431	0.1085
	3	-0.0050	0.06763	1.000	-0.1789	0.1688
2	0	<b>-0.2778*</b>	0.03464	0.000	-0.3669	-0.1887
	1	0.0673	0.06839	0.759	-0.1085	0.2431
	3	0.0622	0.02597	0.078	-0.0045	0.1290
3	0	<b>-0.3400*</b>	0.03312	0.000	-0.4252	-0.2549
	1	0.0050	0.06763	1.000	-0.1688	0.1789
	2	-0.0622	0.02597	0.078	-0.1290	0.0045

Based on observed means.

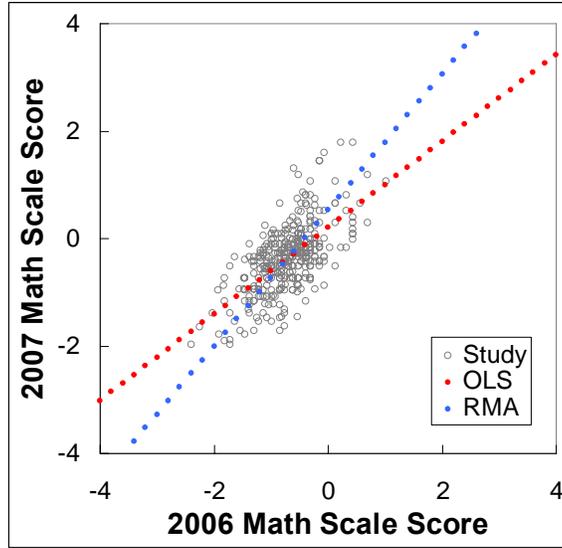
\* indicates mean difference is significant at the 0.05 level.

**Table 3. Tukey Honestly Significant Different post hoc test**

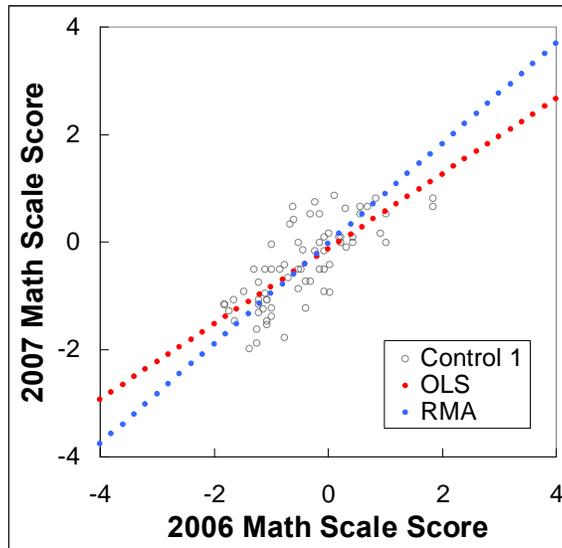
Only the Study group showed a significant difference, indicating that membership in the Study group affected student scores.

### ***OLS and RMA Regression***

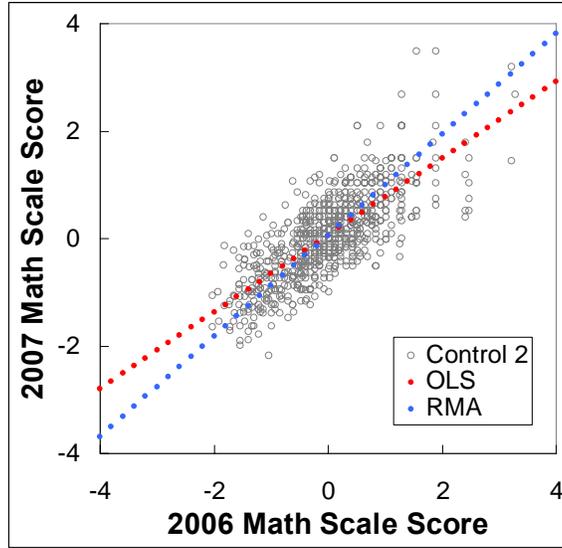
A comparison of OLS to RMA regression for the different classification of students was done as shown in Figures 1 through 4. In every instance, the RMA best fit line bisected the data more evenly than the OLS best fit line. OLS underestimated the slope of the best fit line as compared to RMA. This resulted in an underestimation of predicted student performance at the high end while overestimating performance at the low end.



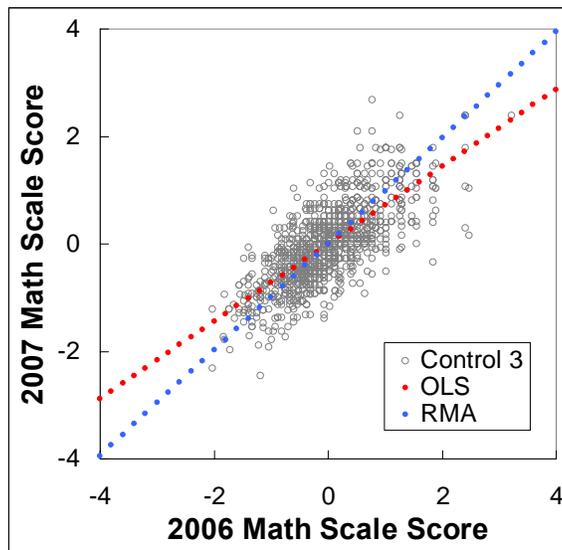
**Figure 1. 2007 score as a function of 2006 score for Study students**  
 (OLS:  $y = 0.8041x + 0.1984$ , RMA:  $y = 1.2665x + 0.5249$ ,  $N = 370$ )



**Figure 2. 2007 score as a function of 2006 score for Control 1 students**  
 (OLS:  $y = 0.6987x - 0.1378$ , RMA:  $y = 0.9317 - 0.0380$ ,  $N = 70$ )

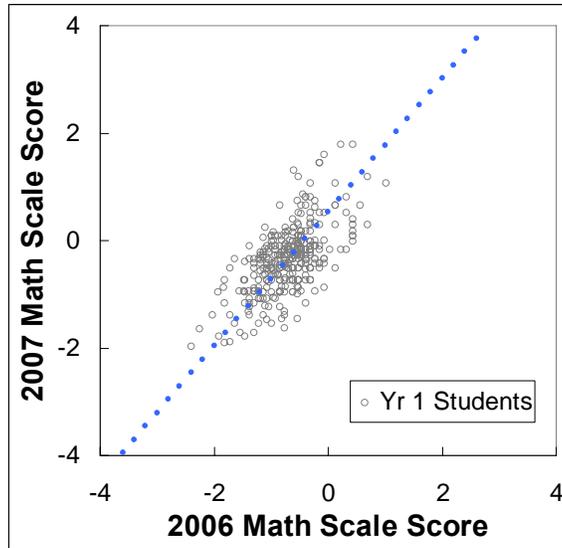


**Figure 3. 2007 score as a function of 2006 score for Control 2 students**  
 (OLS:  $y = 0.7144x + 0.0582$ , RMA:  $y = 0.9382x + 0.0588$ ,  $N = 772$ )

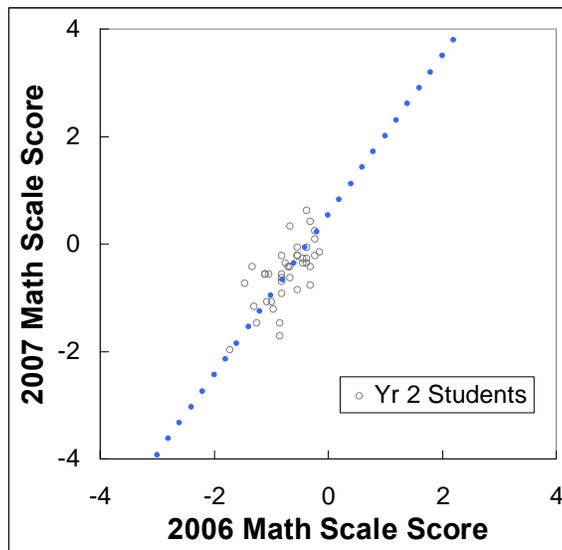


**Figure 4. 2007 score as a function of 2006 score for Control 3 students**  
 (OLS:  $y = 0.7198x - 0.0060$ , RMA =  $0.9859x - 0.0035$ ,  $N = 1,051$ )

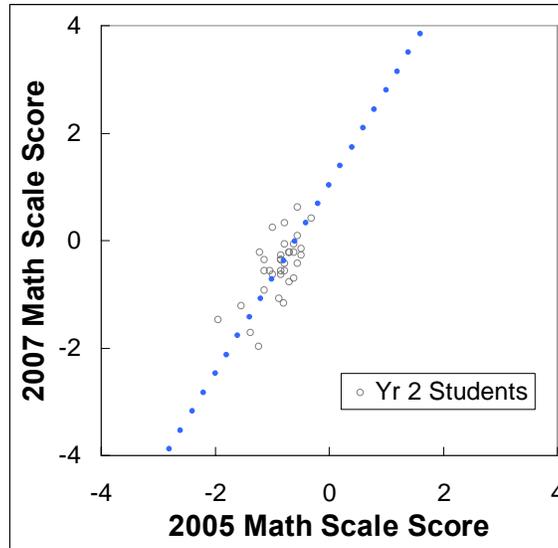
Longitudinal effects of being in the MathForward program were also revisited as shown in Figures 5 – 7. Only RMA regression was done on these figures.



**Figure 5. 2007 scale score as a function of 2006 scale score for first year MathForward students ( $y = 1.2444x + 0.5282$ ,  $N = 330$ )**



**Figure 6. 2007 scale score as a function of 2006 scale score for second year MathForward students ( $y = 1.4844 + 0.5257$ ,  $N = 40$ )**



**Figure 7. 2007 scale score as a function of 2005 scale score for second year MathForward students ( $y = 1.7584x + 1.0367$ ,  $N = 32$ )**

## DISCUSSION

It was shown by the ANOVA and the post hoc test that being in the Study group affected student scores. Furthermore, none of the Control groups were significantly different from each other. The study group showed a mean 0.34 standard deviation unit improvement relative to the testing population between 2006 and 2007 while the Control groups had almost no change in their relative placement. This confirmed the original analysis that the MathForward program was effective at raising student scores in the TAKS Math section.

RMA regression analysis yielded a clearer picture of how this intervention affected students. When the RMA slopes and intercepts were tabulated as shown in Table 4, it was clear that the Study group had a slope that was clearly larger than one while the Control groups had slopes of about 1 or less. Similarly, the Study group had an intercept of about 0.5 standard deviation units above the mean while all the Control groups had intercepts of about 0.

It is important to understand how the slope and intercept differentially affects students. When the slope is equal to 1, the 2006 standard score is a perfect indicator of the 2007

standard score within some margin of error. When the slope is greater than 1, students who scored above a 2006 standard score of 0 increased their 2007 standard score while students with a 2006 standard score of less than 0 decreased their 2007 standard score. The reverse is true when the slope is less than 1. While the slope affects students of different scores differentially, the y-intercept is a consistent modifier of student performance regardless of their 2006 standard score. A positive y-intercept means that every student increased their scores and vice versa for a negative y-intercept.

<b>GROUP</b>	<b>SLOPE</b>	<b>INTERCEPT</b>
<b>Study</b>	1.2665	0.5249
<b>Control 1</b>	0.9317	-0.0380
<b>Control 2</b>	0.9382	0.0588
<b>Control 3</b>	0.9859	-0.0035

**Table 4. Slopes and Y-Intercepts of the Different Groups of Students**

For the Control groups with slopes about 1 and intercepts of about 0, the 2006 standard score was a perfect indicator of the 2007 standard score with no vertical translation necessary. For the Study group, the intercept of about 0.5 standard deviation units meant that all students showed an increase in their scores. However, the slope of about 1.2 meant that students with higher scores benefited more than students with lower scores

Slopes and intercepts for longitudinal effects were tabulated in Table 5. The fact that the intercepts for year to year change for both first and second year MathForward program students was about the ( $\sim 0.5$ ), it meant that students improved each year they were in the program. When looking at the cumulative effects of the second year students, the intercept was about 1 standard deviation unit or approximately the sum of each year's benefit. The slope, which shows how the intervention program affected students of different achievement level differentially, increased with each year in the program. This meant that higher achieving students derived more and more benefit from being in the intervention program as compared to lower achieving students.

<b>YEAR</b>	<b>SLOPE</b>	<b>INTERCEPT</b>
<b>One 06-07</b>	1.2444	0.5282
<b>Two 06-07</b>	1.4844	0.5257
<b>Two 05-07</b>	1.7584	1.0367

**Table 5. Slopes and Y-Intercepts of the different years of students**

The slope of the program's intervention effect across both years was approximately the product of each year's slope. All cumulative effect results should be regarded with some caution due to the fact that the 2005 implementation of the MathForward program used a pullout selection process that could bias longitudinal values and also that the N values were on the low side.

## **CONCLUSIONS**

Reduced Major Axis regression is an error-in-variables model that is useful in cases where both independent and dependent variables are assumed to have measurement error, which is usually the case of a repeated measures study. Based on these new analysis using RMA regression, it was concluded that students in the MathForward program between 2005 and 2007 increased by about half a standard deviation unit for each year they were enrolled in the program. This increase, however, is tempered by their prior performance with higher scoring students deriving greater benefit while under performing students deriving less benefit. These effects accumulate for each year the student was enrolled in the program.

Furthermore, based on the no effect results of the Control 1 group who were students taught by the intervention program teachers but were not in the program, it was concluded that there were aspects of the program that could not be replicated outside of the intervention. Program teachers received professional development in order to be able to execute the intervention. Since this professional development did not affect their students who were not in the program, this could be an indicator that the TI Navigator™ system was an integral part of the intervention to raise student scores.